Measurements of Mach 3 Turbulence Transport Properties on a Nozzle Wall

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Boundary-layer measurements with a Laser Doppler Velocimeter (LDV), using the differential Doppler mode, have been made in the Naval Surface Weapons Center boundary-layer channel at a nominal Mach number of 3. Statistical techniques were used to obtain the various flow parameters which include mean velocity profiles, turbulence intensities, Reynolds shear stress distributions, mixing length, eddy viscosity, and also skewness and flatness measurements of the velocity distribution function. Measurements have also been made of the mean flow properties with conventional pitot-temperature probes. These measurements have been used to compute the various transport properties. These values then are compared with the LDV measurements. The agreement, in general, is found to be quite good.

Nomenclature

aw	= adiabatic wall data, $T_w/T_{aw} = 1.0$
c_p	= specific heat at constant pressure
ℓ	= mixing length
M_∞	= freestream Mach number
N	= number of discrete velocities in ensemble
p	= pressure
Pr_t	= turbulent Prandtl number
q	= total heat transfer
R_{uv}	= shear stress correlation coefficient
Re_θ	= momentum thickness Reynolds number
T	= temperature
u,v	= x and y components of velocity
U	= instantaneous flow velocity in x direction
$\langle u' \rangle$	= rms of velocity in x direction
U_L	= local mean velocity in x direction
U_∞	= mean freestream velocity in x direction
$\langle v' \rangle$	= rms of velocities in y direction
X	= direction parallel to flat plate
Y	= direction normal to flat plate
δ	= boundary-layer thickness
δ_i^*	= incompressible boundary-layer displacement thickness
ϵ	= kinematic eddy viscosity
ϵ	= kinematic eddy conductivity
ρ	= density
τ	= shear stress
σ	= standard deviation
θ	= compressible boundary-layer momentum thickness

Superscripts

$\overline{(\ )}$	= time average
$(\ ')$	= fluctuating quantity

Subscripts

0	= tunnel supply conditions, total
t	= turbulent
w	= wall

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Introduction

SINCE the first measurements with the Laser Doppler Velocimeter (LDV) were reported a decade ago, a considerable amount of effort has gone into developing the LDV into a viable instrumentation system which the fluid dynamicist could use to make measurements, some of which have been exceedingly difficult with conventional techniques. This is particularly true when one is trying to make turbulence measurements in a supersonic boundary layer. The LDV is especially suited for turbulence measurements, since it can make instantaneous measurements of the velocity without disturbing the flow. The primary requirement is that micron-size particles be in the flow. Laser radiation scattered from these particles experiences a frequency or Doppler shift which is proportional to the velocity. A set of discrete velocity data points then must be processed statistically to obtain the desired fluid-dynamic properties. This paper describes a study carried out in the Naval Surface Weapons CIenter boundary-layer channel[1] at Mach 3 to investigate in considerable detail the turbulence properties of a supersonic turbulent boundary layer on the nozzle wall. Measurements also were made with conventional pitot-temperature probes in order to obtain mean flow distributions through the boundary layer. These mean flow data and the time-averaged conservation equations were used to calculate the turbulent flux terms, such as the mixing length and turbulent shear stress.[2] These results then are compared with the limited amount of available data.

Analysis

The evaluation of the Mach 3 turbulent boundary-layer flow makes use of the turbulent flow equations developed by van Driest[3] for compressible fluids. These equations relate the shear stress and heat-transfer rate to the time-averaged quantities in the flow. The following expressions were used to compute the total shear stress and total heat transfer for the two-dimensional nozzle wall flow:

$$\tau = \tau_w + \int_0^y \frac{\partial \bar{p}}{\partial x}\,dy + \int_0^y \frac{\partial}{\partial x}\,[\bar{u}(\bar{\rho}\bar{u})]\,dy - \bar{u}\int_0^y \frac{\partial}{\partial x}\,(\bar{\rho}\bar{u})\,dy \tag{1}$$

and

$$q = q_w - \bar{u}\tau + c_p\left[\int_0^y \frac{\partial}{\partial x}\,(\bar{T}_0\bar{\rho}\bar{u})\,dy - \bar{T}_0\int_0^y \frac{\partial}{\partial x}\,(\bar{\rho}\bar{u})\,dy\right] \tag{2}$$

The turbulent shear stress and heat transfer are simply the difference between the total and laminar values, respectively, where

$$\tau_t = \tau - \delta_\ell,\ q_t = q - q_\ell \tag{3}$$

The turbulent transport terms, namely, the mixing length, turbulent eddy viscosity, and turbulent Prandtl number, were computed from the following relations based on mean flow properties:

$$\tau_t = \bar{\rho}\ell^2\, |d\bar{u}/dy|\, (2\bar{u}/dy) \tag{4}$$

$$\tau_t = \bar{\rho}\epsilon\,(\partial\bar{u}/\partial y) \tag{5}$$

$$q_t = -c_p \rho \overline{v'T'} = \bar{\rho}\epsilon_h\,(\partial T/\partial y) \tag{6}$$

$$Pr_t = \epsilon/\epsilon_h \tag{7}$$

The LDV, which measures the instantaneous flow velocity, is used to obtain only the first term of the following expression for the turbulent shear stress:

$$\tau_t = -\bar{\rho}\overline{u'v'} - \bar{u}\overline{\rho'v'} - \bar{v}\overline{\rho'u'} - \overline{\rho'u'v'} \tag{8}$$

The term $\overline{u'v'}$ is determined statistically from the LDV measurements, as will be discussed later. For incompressible boundary-layer flows, it can be shown that the last three terms of Eq. (8) are negligible compared to the first. This simplification has been extended for compressible flows, although its validity has not been proven completely. Further description of the present method of obtaining the turbulent transport terms from the time-average mean flow measurements and its sensitivity to data spacing is given in Ref. 2.

Experiment

Extensive experimental data were obtained in the Naval Surface Weapons Center Boundary-Layer Channel facility described in Ref. 1. This facility is a continuous blowdown wind tunnel designed specifically for boundary-layer research. The conventional symmetrical supersonic nozzle has been replaced by a nozzle having a flat test plate as one wall, a flexible plate as the opposite wall, and two slightly diverging sidewalls (see Fig. 1). The desired distribution of local flow properties along the test plate can be achieved by adjusting the flexible plate contour. Measurements taken on the flat test plate included LDV profiles, pitot pressure and total temperature profiles, skin-friction drag, and local heat-transfer rates. Detailed descriptions of the facility and its specialized boundary-layer instrumentation can be found in Refs. 1 and 4-8.

A schematic of the LDV system used in the present investigation is shown in Fig. 2 and has been described in Refs. 7 and 8. This type of LDV commonly is referred to as the dual scatter or differential Doppler system. An argon laser operating at 5145 Å, with a nominal output power of 1W, is used as the primary laser source. The scatter volume or focal volume of the system is approximately 1 in. long, with a diameter of 0.015 in. The collecting optics are placed off-axis at an angle of 10°, reducing the scattering volume length to less than 0.1 in. Particles which pass through the focal volume scatter a portion of the laser radiation. This scattered radiation which now has been Doppler shifted is detected by the photomultiplier and then is processed by a digital data processor to determine the Doppler frequency of the radiation scattered by individual micron-size particles. Typical Doppler frequencies are of the order of 40 MHz.

In order to obtain the mean values and the other turbulence parameters, one must analyze the data statistically.[8] If V_i is the instantaneous measurement of velocity, and N measurements are obtained, then the mean velocity and the

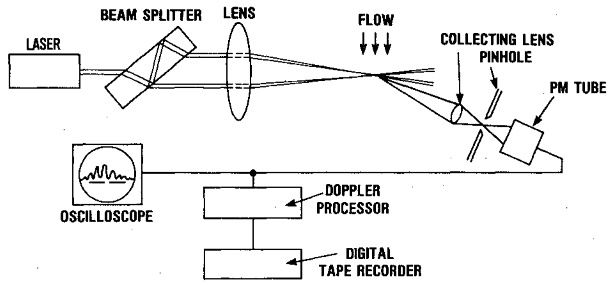

Fig. 1 Boundary-layer channel.

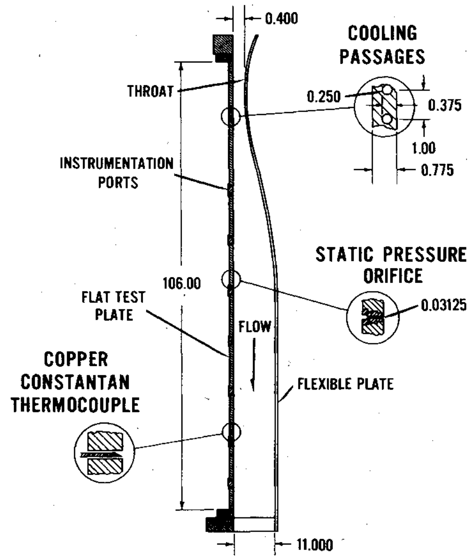

Fig. 2 LDV schematic (dual scatter).

standard deviation are given by

$$\bar{V} = \Big(\sum_{i=1}^{N} V_i/N,\ \langle V' \rangle = \Big[\Big(\sum_{i=1}^{N} (V_i - \bar{V})^2\Big)/(N-1)\Big]^{1/2} \tag{9}$$

where $V = \bar{V} + V'$.

Other parameters calculated are the moment coefficient of skewness, where

$$\text{skewness} = \Big(\sum_{i=1}^{N} (V_i - \bar{V})^3\Big)/N\sigma^3 \tag{10}$$

and the flatness factor, where

$$\text{flatness} = \Big(\sum_{i=1}^{N} (V_i - \bar{V})^4\Big)/N\sigma^4 \tag{11}$$

The mean and standard deviations were computed, and then all data greater than 3σ from the mean were rejected. The remaining data then were used to recompute the mean and standard deviations and other moments. Approximately 4500 data points were taken at each location in the boundary layer. This corresponds to a statistical error of less than 1% in the local mean velocity with a 95% confidence level. This also corresponds to a 2% error in the standard deviation with a 95% confidence level. In determining the turbulent or Reynolds shear stress, two additional surveys were taken with the optics rotated at ±45° to the flow.[8] Approximately 4500 data points were taken at each location in the profile for each survey; thus, each local value of the shear stress is determined from approximately 13,500 data points.

Measurements were obtained for an adiabatic wall condition where the supply temperature was 580°R. The LDV measurements have been obtained for two supply pressures: 1

Table 1 Test conditions

Station	57	67	75	81	87
p_0, psi	15.2	15.2	15.2	15.2	15.0[a]
T_0, °R	582	584	565	576	586[a]
M_∞	2.94	2.93	2.91	2.91	2.93[a]
$Re_\theta \times 10^{-3}$	10.3	11.3	13.1	13.9	13.6[a]
δ, in.	0.890	1.043	0.968	1.057	1.096[a]
δ_i^*, in.	0.115	0.130	0.141	0.153	0.158[a]
θ, in.	0.056	0.062	0.067	0.074	0.076[a]
for $p_0 = 15$ psia, $T_0 = 580$°R, and $M_\infty = 2.9$:					
δ, in.	...	0.93	0.94	0.97	1.08[b]
δ_i^*, in.	...	0.133	0.141	0.153	0.158[b]

[a] Mean flow.
[b] LDV.

and 2 atm, which correspond to Reynolds numbers per foot of 2.2×10^6 and 4.4×10^6, respectively. The mean flow probe measurements were obtained at a supply pressure of 1 atm only (see Table 1). The present paper will discuss only the 1-atm data, since the results at 2 atm are similar. Results for the 2-atm data can be found in the original paper. Measurements were made along the nozzle wall from 60 in. to 90 in. downstream of the nozzle throat. All measurements were made in a zero-pressure gradient region.

Because the particle dynamics are of vital importance in LDV measurements, a large effort was devoted to determining the effects of particle size on the turbulence measurements.[9] The optical volume first was fixed at a distance of 0.300 in. from the wall. In this case three different aerosols were used. The first two were generated from dioctylphthalate (DOP), and had diameters of 1 and 5 μ, respectively. These aerosols were generated using a Laskin nozzle and a single-stage cascade impactor.[7] The third aerosol was generated from water vapor which was injected into the nozzle and formed ice clusters. The mean size of the ice particles was determined, from independent tests, to be less than 0.7μ. These aerosols were injected at various rates into the flow. It was found that if the injection level was doubled, the measured turbulence intensity $\langle u' \rangle / U_L$ increased from 0.072 to 0.074 for the three aerosols; thus, there was negligible effect caused by the rate of injection.

Profiles also were taken with different aerosols. In this case DOP was used, and the aerosols again had nominal diameters of 1 and 5 μ. There was very little difference in the results between the two sizes. The maximum velocity difference was 13 fps, and the maximum intensity difference was 0.3%. As a consequence, the larger aerosol was used for all of the final boundary-layer profile measurements. The large size had the distinct advantage in that the signals obtained from the 5 μ particles are more than 20 times greater than the signals from the 1μ particles. This is due to the Mie scattering dependence on the particle diameter.

Results

In Fig. 3 the mean velocity profiles obtained at four stations along the test plate are shown. Also shown in Fig. 3 are the velocities obtained with conventional pitot and total temperature probes (solid lines).[10] Each probe profile consists of about 50 data points with nonuniform spacing. The distance between profile points varies from 0.001 in. near the wall to 0.1 in. near the freestream. The agreement between the two techniques is quite good.

Shown in Fig. 4 are the turbulence intensity distributions. Also shown are the data from Sandborn's review[11] of compressible turbulence measurements. This includes the incompressible data of Klebanoff[12] and the LDV data of Rose and Johnson[13] which were obtained on a nozzle wall at Mach 3, Kistler's[14] data, and Markovin and Phinney's[15] data. It is immediately evident that the incompressible and compressible turbulence intensities are very similar.

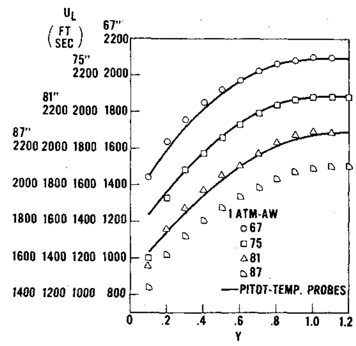

Fig. 3 Mean velocity profiles along plate.

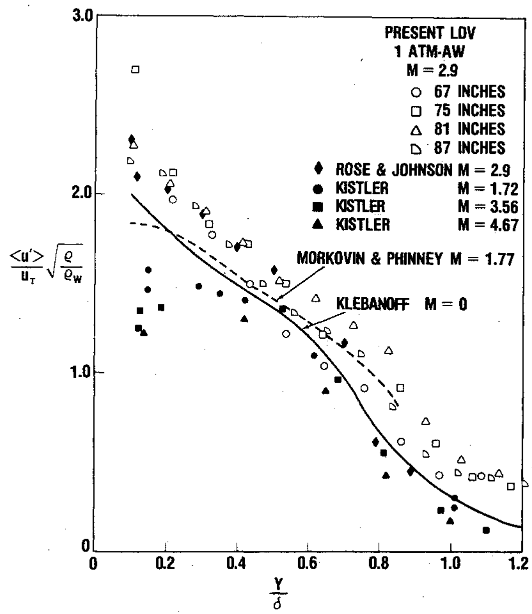

Fig. 4 Turbulence intensity distributions (longitudinal).

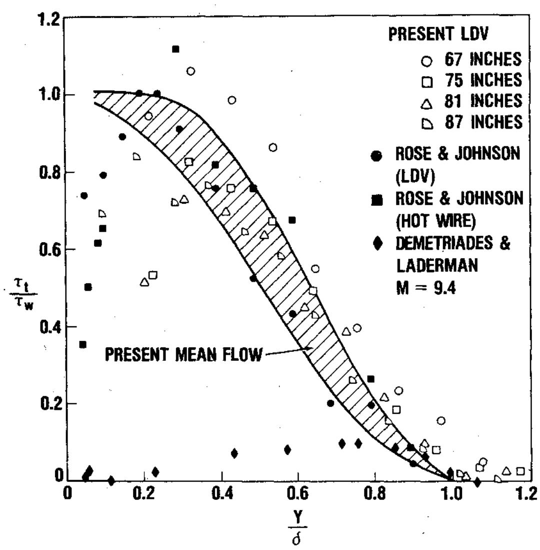

Fig. 5 Reynolds shear stress distribution.

In Fig. 5 are shown the Reynolds shear stress distributions, along with the data from Fig. 3 of Ref. 11. These are referenced to the wall shear stress. It is here in the turbulent shear stress that one sees some discrepancy between the two methods. The LDV data indicate a drop in shear stress near the wall much sooner than the mean flow data indicate. However, the present LDV data are in agreement with the data of Rose and Johnson which were obtained with both the

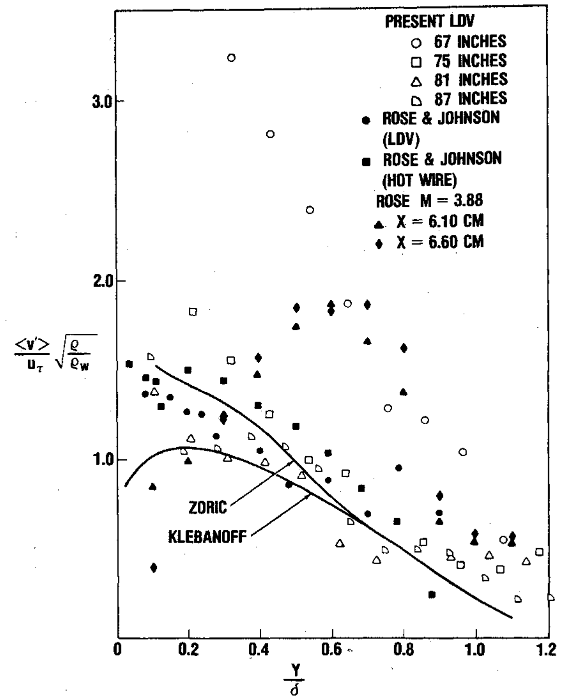

Fig. 6 Turbulence intensity distributions (lateral).

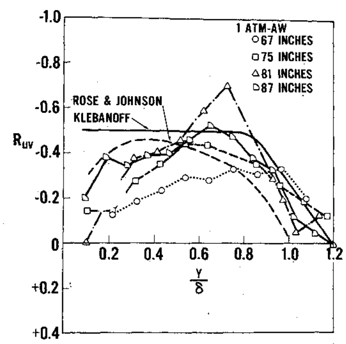

Fig. 7 Turbulent correction coefficient distributions.

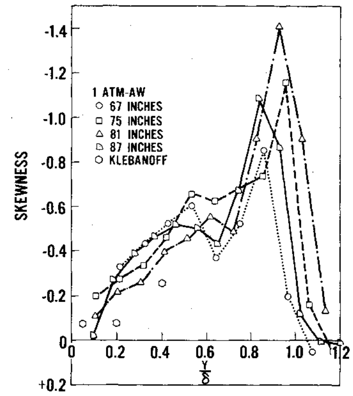

Fig. 8 Coefficient of skewness distributions.

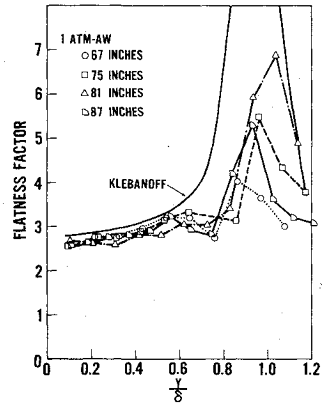

Fig. 9 Flatness factor distributions.

LDV and the hot-wire anemometer. One explanation may be the fact that density fluctuations may not have been accounted for completely. It has been assumed tacitly that the turbulent shear is equal to $\rho u'v'$, with the other terms of the shear stress being negligible. This may not be the case and warrants further investigation.

The lateral turbulence intensity distributions are shown in Fig. 6 along with the data from Fig. 12 of Ref. 11. There is more scatter in these measurements, because $\langle v' \rangle$ is an indirect measurement, since it relies on three independent measurements. However, there is still good agreement with the data of Klebanoff and with Rose and Johnson. One set of data (1 atm-67 in.) is seen to be considerably higher. The data were repeatable, and no reason was found for the discrepancy.

The turbulent shear correlation coefficient distributions are presented in Fig. 7, where

$$R_{uv} = \overline{u'v'}/(\langle u' \rangle \langle v' \rangle) \tag{12}$$

Again, there is good agreement with the available data, although, in general, the compressible data tend to lie below the incompressible data. The decrease in the value of the shear in the wall region is evidenced immediately in the falloff of the correlation coefficient.

The results for the moment coefficient of skewness (third moment of the velocity probability distribution function) are shown in Fig. 8. A negative skewness indicates a skewness of the velocity probability distribution function toward velocities less than the mean. It is interesting to note that the skewness has a dip in the region where y/δ is about 0.7. This was characteristic of all of the distributions, including the flatness factor (fourth moment) shown in Fig. 9. A gaussian distribution has a flatness factor equal to 3. The data are plotted in terms of the flatness factor rather than the intermittency factor. Shown also are the available data of Klebanoff. The flatness and the skewness data show a large prominence near the boundary-layer edge because of the intermittent flow boundary. The present data have the same trends, although they are lower in general than Klebanoff's data, thus indicating that the intermittent nature of a supersonic boundary layer is very similar to an incompressible boundary layer.

The Prandtl mixing length distribution for both LDV and mean flow measurements at the 1-atm supply pressure are shown in Fig. 10. The scatter of the data at the outer edge of the boundary layer, i.e., $y/\delta > 0.8$, is caused by the sensitivity of the computational method to the velocity derivative, which is very small in this region. Good agreement is seen between the two techniques. The results of Maise and McDonald[16] showed this correlation of the mixing length to be insensitive to both Mach number and Reynolds number for the flat-plate adiabatic wall flow. The mixing length results from the mean flow method are compared with the analytical results of Maise-McDonald and Economos-Boccio[17] and with the results obtained in the same facility at Mach number 5.[2] It was demonstrated in Ref. 2 that the upstream cooling of the nozzle wall boundary layer in the throat region has the effect of decreasing the plateau level of the mixing length value at midboundary layer. The present Mach number 3 condition still contains some upstream cooling at the nozzle throat region but not as severe as that used at the Mach number 5 condition. This upstream cooling effect also is reflected in the present data. It is conjectured, at this point, that the mixing length distribution on the nozzle wall would be that of flat-plate values if the wall were completely adiabatic. The kinematic eddy viscosity distributions, obtained from both the LDV and mean flow measurements, are shown in Fig. 11.

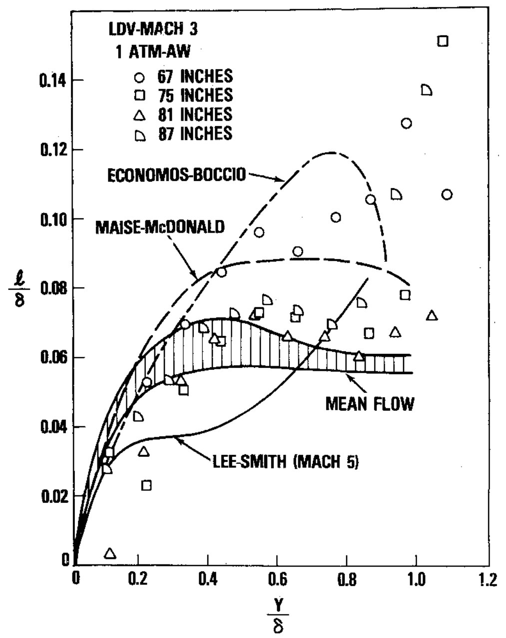

Fig. 10 Mixing length distributions.

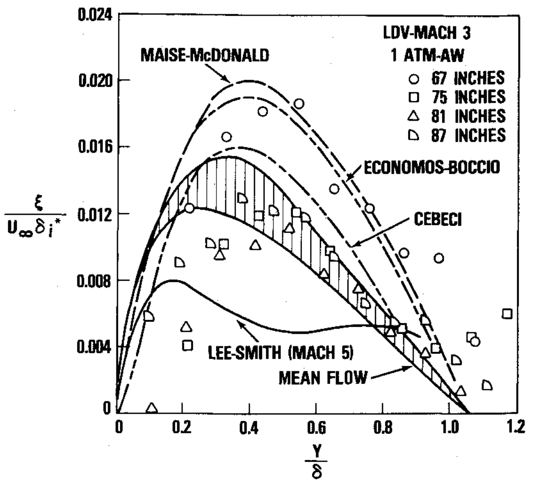

Fig. 11 Kinematic eddy viscosity distributions.

With the exception of the LDV data measured at the 67-in. station, the LDV results in the outer region of the boundary layer agreed very well with the mean flow results, the upper and lower bounds of which are indicated by the two solid curves. The disagreement at the inner portion of the boundary layer has been described in the discussion of the shear stress distribution. The LDV measurements at the 67-in. station are consistently higher and repeatable, and no reason is found to support this inconsistency. The comparison of the eddy viscosity distributions between the mean flow measurements and three flat-plate flow predictions[16-18] is shown in Fig. 11. Also shown for comparison are the Mach number 5 nozzle wall results reported in Ref. 2. Similar to the mixing length comparison, the present Mach number 3 data still retain some upstream cooling influence.

Conclusions

The turbulent transport properties of a Mach 3 turbulent boundary layer with zero pressure gradient have been investigated. A Laser Doppler Velocimeter, together with mean flow measurements obtained with conventional probes, has been used to measure and evaluate the various turbulence parameters. Measurements include mean velocity profiles, turbulence intensities, Reynolds shear stress, mixing length, eddy viscosity, and turbulent Prandtl number distributions and also skewness and flatness measurements of the velocity probability distribution function. The following conclusions were reached:

1) Agreement between the LDV and mean flow measurements of the various turbulence parameters was quite good.

2) Some of the turbulence parameters measured bear a remarkable similarity to incompressible data not only in trends but also in magnitude, thus indicating a Mach number independence. This is particularly true for the turbulence intensity and the skewness.

3) There was some discrepancy in the Reynolds shear stress determined by LDV and the mean flow measurements. The LDV measurements were consistently lower in the last 30% of the boundary layer near the wall. It is a speculation at this point that the Reynolds stress as determined from $\rho \overline{u'v'}$ may not be complete, and that density fluctuations may be of significance. This will need further evaluation.

4) The data measured reflect the effect of upstream boundary-layer cooling on the transport properties. This is consistent with previously reported nozzle wall results.

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